

1.2 Polynomials

Question Paper

Course	CIEAS Maths
Section	1. Algebra & Functions
Topic	1.2 Polynomials
Difficulty	Very Hard

Time allowed: 80
Score: /63
Percentage: /100

Question 1a

(a) Expand and simplify $(x + y)(x - y)(y - x)(-x - y)$.

[2 marks]

Question 1b

(b) A cuboid has a length of $(2x - 3y + 3)$ units, a width of $(2x + 3y - 3)$ units, and a height of $(x - y)$ units. Find an expression for the volume of the cuboid in terms of x and y .

[2 marks]

Question 2

Given that $(ax + by)(2x + y)(x - 3y) = 8x^3 + cx^2y + dxy^2 - 9y^3$, where a, b, c and d are constants, find the values of a, b, c and d .

[3 marks]

Question 3

Factorise completely $x^5y - xy^5$.

[3 marks]

Question 4

Divide $4x^4 - 37x^2 + 9$ by $(2x - 1)$.

[3 marks]

Question 5a

$$f(x) = 6x^4 + 7x^3 - 27x^2 - 28x + 12$$

(a) Find the remainder when $f(x)$ is divided by $(2x + 3)$.

[2 marks]

Question 5b

(b) Given that $(x + 2)$ is a factor of $f(x)$, factorise $f(x)$ completely.

[5 marks]

Question 6a

$$f(x) = 3x^4 + x^3 - 12x^2 - 49x - 15$$

(a) Show that $f(x) = (3x + 1)(ax^3 + bx^2 + cx + d)$ where a, b, c and d are constants to be found.

[2 marks]

Question 6b

(b) Given that $(x - 3)$ is a factor of $f(x)$, factorise $f(x)$ completely.

[5 marks]

Question 6c

(c) Hence show that the equation $f(x) = 0$ has exactly 2 real roots.

[2 marks]

Question 7

Given that 3 is a root of the equation $2x^3 - x^2 - 11x - 12 = 0$, prove that the equation has no other real roots.

[4 marks]

Question 8a

$$f(x) = 2x^4 - 15x^3 - 10x^2 + 105x + 98$$

(a) Show that $f(-1) = 0$ and $f(-2) = 0$.

[1 mark]

Question 8b

(b) Hence, solve $f(x) = 0$.

[7 marks]

Question 9

Given that $(2x - 5)$ is a factor of the function

$$f(x) = 2x^3 + kx^2 - 11x - 60$$

find the value of k and fully factorise $f(x)$.

[4 marks]

Question 10

Show that $(9x^2 - 4)$ is a factor of $9x^4 - 40x^2 + 16$ and hence find all the real solutions to the equation $9x^4 - 40x^2 + 16 = 0$.

[5 marks]

Question 11a

(a) Show that $(ax - 2)$ is a factor of $3ax^2 + (a - 6)x - 2$.

[2 marks]

Question 11b

(b) Given that $x = -\frac{1}{a-4}$ is a root of $3ax^2 + (a-6)x - 2$, find the value of a .

[3 marks]

Question 12a

For a polynomial $f(x)$, the Remainder Theorem states that

When $f(x)$ is divided by $(ax - b)$ the remainder is $f\left(\frac{b}{a}\right)$.

(a) Use the Remainder Theorem to find the remainder when $8x^3 + 6x^2 - x - 2$ is divided by $(2x + 1)$.

[2 marks]

Question 12b

(b) Work out the remainder when $6x^2 - x - 2$ is divided by $(2x + 1)$.

[2 marks]

Question 13

When $2x^3 + (a + b)x^2 + (a - b)x - 3$ is divided by $x + 4$ the quotient is $2x^2 + (2a + 3)x + (2b - 5)$ and the remainder is c .

Find the values of a, b and c .

[4 marks]